Robust Orientation Estimation with TRIAD-aided Manifold EKF

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Motivation

- Stabilisation of a Quadcopter under harsh initialisation - **TeensyPilot**
- Needs Orientaion Estimation: A very well-studied problem.
- Using MARG sensor stack on aerial vehicles.
- **Magnetometer** to tackle drift in the yaw axis.
    - Prone to disturbances - sensitivity to calibration.
    - Disturbs the roll and pitch estimates (critical for attitude stabilisation)
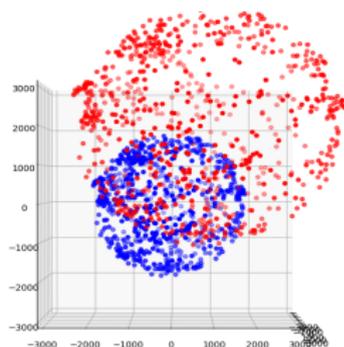
(a) Mag Readings-
Original and Calibrated
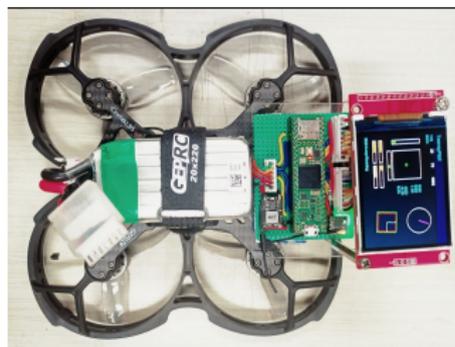
(b) Quadcopter with MARG
Sensor

Background

Algorithm

- E. J. Lefferts, F. L. Markley, and M. D. Shuster,
  "Kalman filtering for spacecraft attitude estimation,"
  Journal of Guidance, control, and Dynamics, vol. 5, no. 5, pp. 417–429, 1982.

- M. W. Mueller, M. Hehn, and R. D'Andrea,
  "Covariance correction step for kalman filtering with an attitude,"
  Journal of Guidance, Control, and Dynamics, vol. 40, no. 9, pp. 2301–2306, 2017.

- P. Bernal-Polo and H. Martínez-Barberá,
  "Kalman filtering for attitude estimation with quaternions and concepts from manifold theory,"
  Sensors, vol. 19, no. 1, p. 149, 2019.

- Y. Ge, P. Van Goor, and R. Mahony,
  "A note on the extended kalman filter on a manifold,"
  62nd IEEE Conference on Decision and Control (CDC) - 2023

Tackling Magnetic Disturbance

- B. Fan, Q. Li, and T. Liu, "How magnetic disturbance influences the attitude and heading in magnetic and inertial sensor-based orientation estimation,"
  Sensors (Switzerland), vol. 18, 1 2018.

- P. Martin and E. Salaun, "Invariant observers for attitude and heading estimation from low-cost inertial and magnetic sensors," 46th IEEE Conference on Decision and Control. IEEE, 2007.

- Y. Chen and H. Rong, "A customized extended kalman filter for re- moving the impact of the magnetometer's measurements on inclination determination," Sensors, vol. 23, 12 2023.

Manifold EKF: Concepts

- Unit quaternion $\in \mathsf{S}^3$ (a manifold)
- "Attitude-error representations"[*] are all **charts** φ for the manifold mapping a point in S^3 to $\mathbf{e} \in \mathbb{R}^3$.
- Kalman Filter evolution happens on a chart
- Deformation of the space is minimal where the chart is centered.
- Deviation $\boldsymbol{\delta}$ from $\bar{q}$ to q, $q = \bar{q} * \boldsymbol{\delta}$
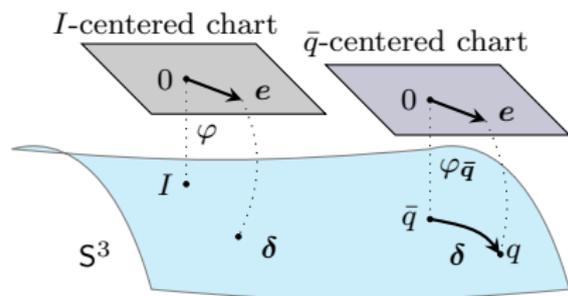
Figure: Charts and Deviation

$$q = \varphi_{\bar{q}}^{-1}(e) = \bar{q} * \varphi^{-1}(e) = \bar{q} * \boldsymbol{\delta}$$

[*]F. L. Markley, "Attitude error representations for kalman filtering," Journal of Guidance, Control, and Dynamics, vol. 26, pp. 311–317, 2003.

Manifold EKF: Prediction

$$\left(\varphi, \quad \bar{q}_{n-1|n-1}, \quad \bar{x}_{n-1|n-1}^{\bar{q}_{n-1|n-1}}, \quad \mathbf{P}_{n-1|n-1}^{\bar{q}_{n-1|n-1}} \right).$$

- chart
- prior estimated q
- prior state estimate and covariance in prior q-centered chart

A priori State Estimate

$$\bar{q}_{n|n-1} = \bar{q}_{n-1|n-1} * \boldsymbol{\delta}(\bar{\boldsymbol{\omega}}_{n|n-1}, \Delta t_n)$$

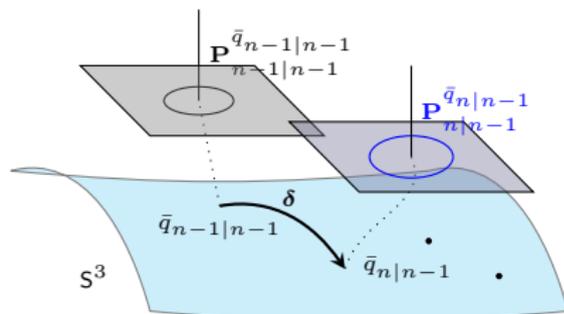$$\bar{\boldsymbol{\omega}}_{n|n-1} = \bar{\boldsymbol{\omega}}_{n-1|n-1}$$

A priori Estimate Covariance

$$\mathbf{P}_{n|n-1}^{\bar{q}_{n|n-1}} = \mathbf{F}_n [\mathbf{P}_{n-1|n-1}^{\bar{q}_{n-1|n-1}} + \mathbf{Q}_n] \mathbf{F}_n^T$$

Measurement Prediction

$$\bar{z}_{a_{n|n-1}} = \mathbf{R}_{\bar{q}_{n|n-1}}^T \mathbf{a_r}$$

$$\bar{z}_{\omega_{n|n-1}} = \bar{\boldsymbol{\omega}}_{n|n-1}.$$

$$\left(\varphi, \bar{q}_{n|n-1}, \bar{x}_{n|n-1}^{\bar{q}_{n|n-1}} = (\bar{e}_{\bar{q}_{n|n-1}}^{\bar{q}_{n|n-1}} = \mathbf{0}, \bar{\boldsymbol{\omega}}_{n|n-1})^T, \mathbf{P}_{n|n-1}^{\bar{q}_{n|n-1}} \right)$$

Manifold EKF: Correction

A **measurement** is made at time nT: $\boldsymbol{z}_n = \begin{pmatrix} \boldsymbol{z}_{a_n} & \boldsymbol{z}_{\omega_n} \end{pmatrix}^T$

Innovation residual

$$\tilde{\boldsymbol{y}}_n = \boldsymbol{z}_n - \bar{\boldsymbol{z}}_{n|n-1}$$

Innovation covariance

$$\mathbf{S}_n = \mathbf{H}_n \mathbf{P}_{n|n-1}^{\bar{\boldsymbol{q}}_{n|n-1}} \mathbf{H}_n^T + \begin{bmatrix} R_a & 0 \\ 0 & R_\omega \end{bmatrix}$$
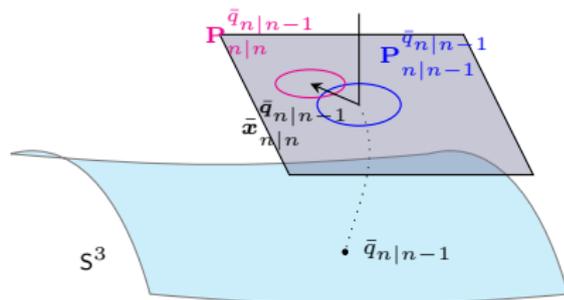
Kalman gain

$$\mathbf{K}_n = \mathbf{P}_{n|n-1}^{\bar{\boldsymbol{q}}_{n|n-1}} \mathbf{H}_n^T \mathbf{S}_n^{-1}$$

A posterior state estimate
and covariance

$$\bar{\boldsymbol{x}}_{n|n}^{\bar{\boldsymbol{q}}_{n|n-1}} = \bar{\boldsymbol{x}}_{n|n-1}^{\bar{\boldsymbol{q}}_{n|n-1}} + \mathbf{K}_n \tilde{\boldsymbol{y}}_n$$

$$\mathbf{P}_{n|n}^{\bar{\boldsymbol{q}}_{n|n-1}} = (I_6 - \mathbf{K}_n \mathbf{H}_n) \mathbf{P}_{n|n-1}^{\bar{\boldsymbol{q}}_{n|n-1}}$$

$$\left(\varphi, \quad \bar{\boldsymbol{q}}_{n|n-1}, \quad \bar{\boldsymbol{x}}_{n|n}^{\bar{\boldsymbol{q}}_{n|n-1}} = (\bar{\boldsymbol{e}}_{\bar{\boldsymbol{q}}_{n|n}}^{\bar{\boldsymbol{q}}_{n|n-1}}, \bar{\boldsymbol{\omega}}_{n|n})^T, \quad \mathbf{P}_{n|n}^{\bar{\boldsymbol{q}}_{n|n-1}}. \right)$$
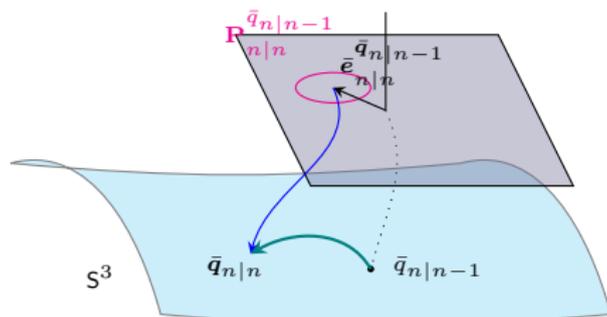
Manifold EKF: Chart Update

Chart Update

$$\bar{q}_{n|n} = \varphi_{\bar{q}_{n|n-1}}^{-1}(\bar{e}_{\bar{q}_{n|n}}^{\bar{q}_{n|n-1}})$$

$$= \bar{q}_{n|n-1} * \varphi^{-1}(\bar{e}_{\bar{q}_{n|n}}^{\bar{q}_{n|n-1}})$$

$$= \bar{q}_{n|n-1} * \bar{\delta}_{\bar{q}_{n|n}}^{\bar{q}_{n|n-1}}$$

Covariance Correction
- using Transition Map: $\mathbf{T}$

$$\mathbf{P}_{n|n}^{\bar{q}_{n|n}} =$$

$$\begin{bmatrix} \mathbf{T}(\bar{\delta}_n) & 0 \\ 0 & I \end{bmatrix} \mathbf{P}_{n|n}^{\bar{q}_{n|n-1}} \begin{bmatrix} \mathbf{T}(\bar{\delta}_n) & 0 \\ 0 & I \end{bmatrix}^T$$

$$\left(\varphi, \quad \bar{q}_{n|n}, \quad \bar{x}_{n|n}^{\bar{q}_{n|n}}, \quad \mathbf{P}_{n|n}^{\bar{q}_{n|n}} \cdot \right)$$

Incorporating the Magnetometer

Magnetometer

Append to Measurement Model:
$$z_{m_t} = \mathbf{R}_{q_t}^T m_r + \mathbf{r}_t^m$$

Include in Measurement Prediction and Innovation Covariance - Correction

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The Challenge

Inconsistency between two reference vectors.

No $\mathbf{R} \in \mathsf{SO}(3)$ can simultaneously satisfy $z_{a_t} = \mathbf{R}^T a_r$ and $z_{m_t} = \mathbf{R}^T m_r$
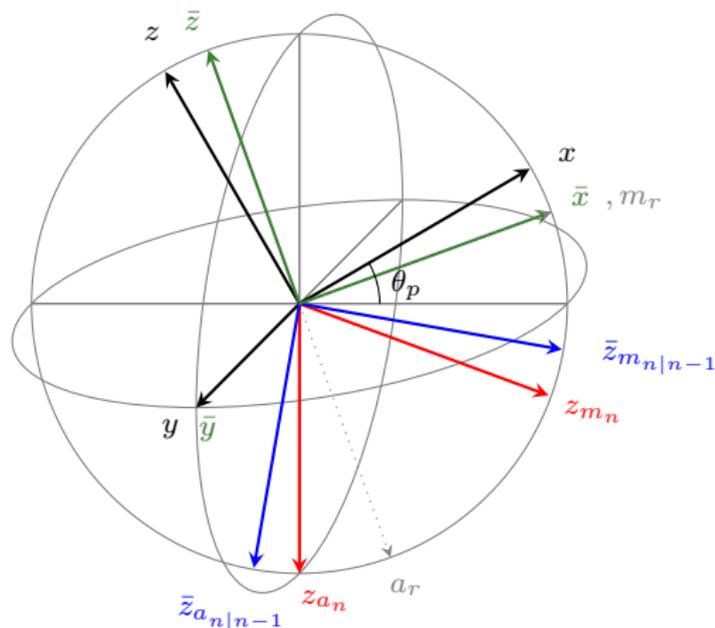

Figure: Effect of Magnetic Disturbances on the Manifold EKF2.

TRIAD aided Manifold EKF

TRIAD:

$\mathbf{c}_{1n} = \boldsymbol{z_{a_n}}$;

$\mathbf{c}_{2n} = \dfrac{\boldsymbol{z_{a_n}} \times \boldsymbol{z_{m_n}}}{|\boldsymbol{z_{a_n}} \times \boldsymbol{z_{m_n}}|}$;

$\mathbf{c}_{3n} = \mathbf{c}_{1n} \times \mathbf{c}_{2n}$

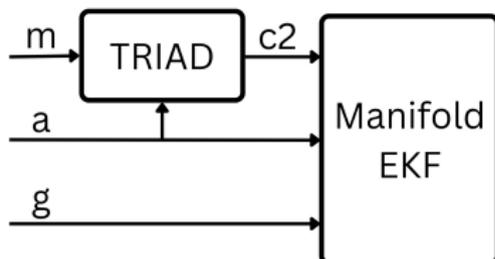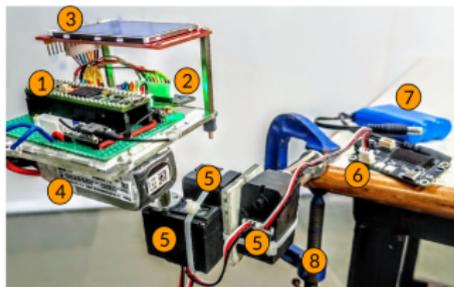$\mathbf{R_n} = \begin{bmatrix} \mathbf{c}_{1n} & \mathbf{c}_{2n} & \mathbf{c}_{3n} \end{bmatrix}$


Figure: Layout of the TRIAD aided Manifold EKF.

- Accelerometer measurement - anchor vector.
- $\mathbf{c}_{2n}$ or $\mathbf{c}_{3n}$ used instead of $\boldsymbol{z_{m_n}}$ in Manifold EKF
- Ignores component of magnetometer that influences attitude (Roll and Pitch).

1) Teensy 4.1 Dev Board
2) LSM9DS1 - 9-axis MIMU
3) TFT Display
4) 3s LiPo Battery
5) Waveshare ST3215 Servo (x3)
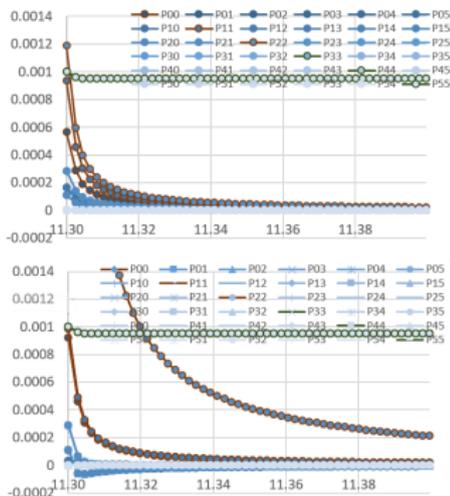6) ESP32 Servo Driver
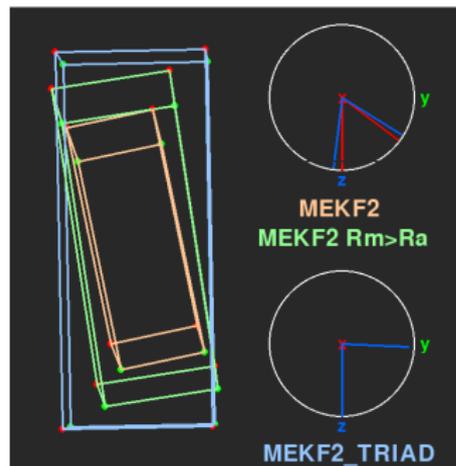7) 3S Li-Ion Battery
8) Clamp

(a) Estimator Test Bench
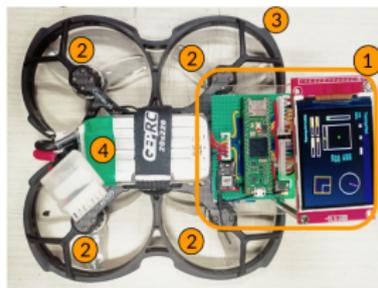
1) Magneto-Inertial
   Measurement Unit (with Display)
2) GepRC SpeedX2
   2105.5 2650kV (x4)
3) GepRC Cinelog 35 V2 Frame
4) 6S LiPo Battery

(b) Quadcopter UAV hardware setup

Qualitative Comparison of Estimators

(c) $\mathbf{P}$ vs time when $R_m = R_a$ and $R_m > R_a$

(d) Cubes and 3D-compasses visualisation for each of the estimators at $90°$ roll.

Modifying noise characteristics may result in sluggish estimator.
TRIAD aid: mag. disturbance decoupled from roll estimate; zero residual;
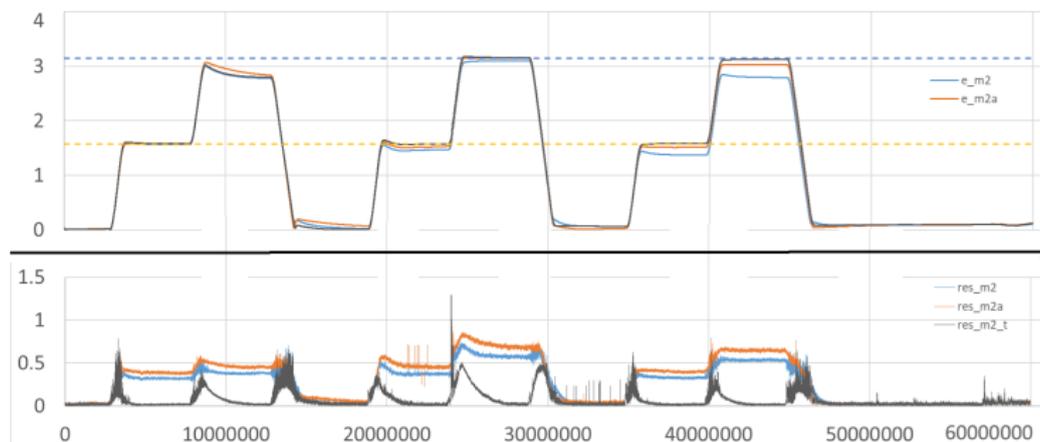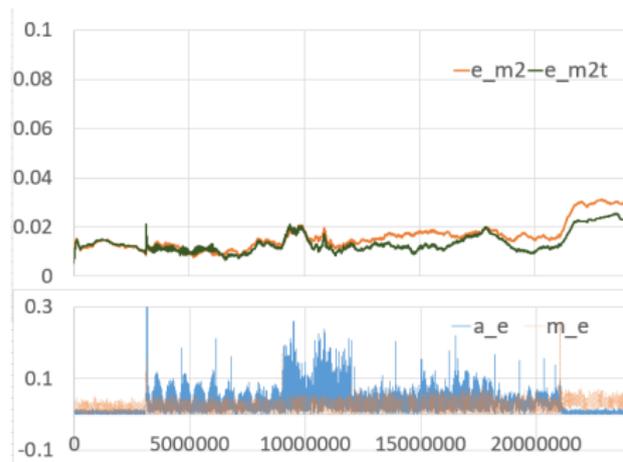
Quantitative Comparison of Estimators

Figure: Rotation Error vs time (top) and norm of the residual vs time (bottom) for the three estimators.
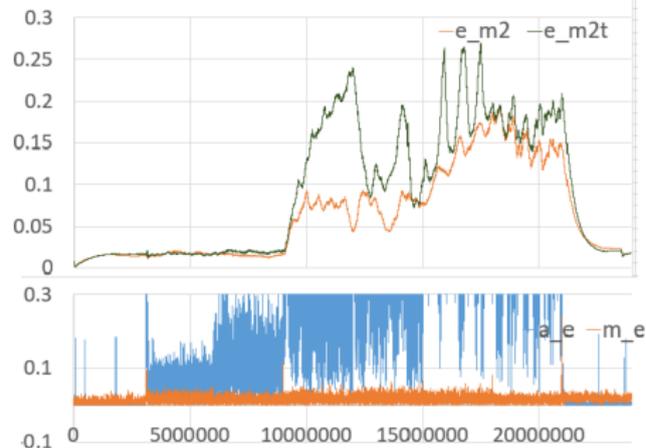
Sequence of standard rotations executed;
At 180° roll: error of 20deg, 6.5deg, .6deg for the three estimators
Residual goes to zero only in the TRIAD-aided case.

Limitations of TRIAD-aid

(a) soft mount   (b) hard mount

Figure: Rotation error in Manifold EKF2 and TRIAD aided Maniold EKF2; accelerometer and magnetometer noises.

Assumption: Accelerometer is reliable.
Fails under large accelerometer noise/disturbances

Thank You

TeensyPilot Github Repo: